

# **NUMERICAL MODELLING OF MAGNETIC MATERIALS FOR COMPUTER AIDED DESIGN OF ELECTROMAGNETIC DEVICES**

by

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## SUMMARY

In design and simulation of electromagnetic devices, it is essential to model the properties of magnetic materials, such as the relation between magnetic flux density  $B$  and magnetic field  $H$  or  $B$ - $H$  curve and electromagnetic power losses or core losses with various kinds of magnetic field excitations, in order to assess the performance correctly.

The major part of the work is concerned with the modelling of hysteresis loops with alternating magnetic field, and core losses with alternating and/or rotating magnetic fields. Various novel models are developed.

A critical comparison between various available models of magnetic hysteresis shows that the Preisach theory appears to be suitable for practical engineering applications. A new normal Preisach model is obtained with the help of a graphical representation of the theory. The new model features simple formulation and easy parameter identification. The input data is the limiting hysteresis loop. It can provide correct results for a medium or large magnetic field, but fails when the hysteresis loop to be predicted is close to the origin of the  $B$ - $H$  plane owing to some intrinsic defects of the model. These defects are eliminated in a new generalised model, which contains a reversible magnetisation component and a magnetisation feed back. The input data required by the generalised model are the limiting hysteresis loop and the normal magnetisation curve. These can be obtained from either manufacturers' data sheets or from simple measurements. Better accuracy is achieved by the generalised model.

New dynamic discrete circuit models with hysteresis, eddy current, and anomalous losses included are developed to simulate the performance of magnetic cores in devices with non-sinusoidal alternating flux. At low frequencies, a simple equivalent circuit model consisting of a constant equivalent resistor for eddy current loss, a nonlinear equivalent resistor for anomalous loss, and a non-ideal inductor for modelling the hysteresis loop and hysteresis loss is used. This model is generalised into a ladder network model for simulation at high frequency by subdividing the cross section of the core into a few assumed eddy current paths. All parameters of these models can be identified from data sheets provided by manufacturers.

For rotational core loss measurement, a single sheet square specimen tester is developed. The precision of two dimensional field strength measurement at the surface of the specimen is improved by a novel sandwich  $H$  sensing coil arrangement. The relationship between the core loss due to the rotational component of magnetic field and the total core loss is clarified using a new equation and the arguments are supported by the experimental results.

Rotational core losses in grain oriented and non-oriented silicon steel sheets were measured using the testers at the University of Technology, Sydney and the Physikalisch-Technische Bundesanstalt, Braunschweig, Germany. These measurements provided much useful information for both understanding of the loss mechanisms and modelling of the losses.

Similar to the case of alternating core losses, rotational core loss can also be separated into rotational hysteresis, eddy current, and anomalous losses. The rotational hysteresis loss is fitted by a novel model based on a strong analogy between the retarding torque due to the rotational hysteresis loss and the electromagnetic torque in a single phase induction machine. With a circular flux density, the rotational eddy current loss is twice as much as the alternating eddy current loss. The rotational anomalous loss can be modelled using the same formula as for alternating anomalous loss, but the coefficient of rotational anomalous loss is generally a function of flux density, and eventually reduces to zero when the material is saturated and all domain walls disappear.

Total core losses with an elliptical flux density are predicted from the pure rotational and alternating core losses by a new formulation derived from the total core loss formula used in rotational core loss measurement. The new model is applicable to hysteresis as well as total core losses. Comparisons with experimental data show that this new model is more accurate than a linear interpolation between alternating and pure rotational core losses.

Core losses in an AC permanent magnet motor are modelled. The magnetic flux density distribution is calculated by a finite element code. Fourier series analysis is used for an arbitrary two dimensional rotating flux density. The total core loss is finally calculated by summing up all the contributions from different elliptically rotating harmonics of flux density in each finite element. The discrepancy between calculated and measured results is about 13%.

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## LIST OF SYMBOLS\*

$A$	Cross sectional area of a magnetic core ( $\text{m}^2$ )
$B$	Magnetic flux density (T)
$B_a$	Alternating component of an elliptically rotating flux density (T)
$B_{\text{maj}}$	Major axis of an elliptically rotating flux density (T)
$B_{\text{min}}$	Minor axis of an elliptically rotating flux density (T)
$B_p$	Peak value of flux density (T)
$B_r$	Rotational component of an elliptically rotating flux density (T)
$B_x$	X component of flux density (T)
$B_y$	Y component of flux density (T)
$B_{\Delta}$	Incremental magnetic flux density (T)
$b$	Thickness of a silicon steel sheet (m)
$C_a$	Alternating anomalous loss coefficient (SI units)
$C_{\text{ar}}$	Rotational anomalous loss coefficient (SI units)
$C_e$	Eddy current loss coefficient (SI units)
$E$	Electric field intensity (V/m)
$f$	Frequency (Hz)
$H$	Magnetic field strength (A/m)
$H_a$	Alternating component of an elliptically rotating field strength (A/m)
$H_b$	Biasing magnetic field strength (A/m)
$H_{\text{maj}}$	Major axis of an elliptically rotating field strength (A/m)
$H_{\text{min}}$	Minor axis of an elliptically rotating field strength (A/m)
$H_r$	Rotational component of an elliptically rotating field strength (A/m)
$H_s$	Magnetic field strength at the surface of a specimen (A/m)
$H_{\text{sat}}$	Saturation magnetic field strength (A/m)
$H_x$	X component of magnetic field strength (A/m)
$H_y$	Y component of magnetic field strength (A/m)
$H_{\Delta}$	Incremental magnetic field strength (A/m)
$I$	Current (A)
$i_a$	Current in an equivalent resistor for anomalous loss (A)
$i_e$	Current in an equivalent resistor for eddy current loss (A)
$i_L$	Current in an inductor (A)
$i_s$	Excitation current (A)
$J$	Current density ( $\text{A}/\text{m}^2$ )
$J_x$	X component of current density ( $\text{A}/\text{m}^2$ )

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\* symbols which are not listed here are defined where they appear.

$K_H$	Coil coefficient of an H sensing coil (SI units)
$L$	Inductance (H)
$l_m$	Mean length of a flux path in an annular ring sample (m)
$M$	Magnetisation (A/m)
$M_{anh}$	An hysteretic magnetisation (A/m)
$M_d(H)$	Magnetisation on the downward trajectory of the limiting hysteresis loop for a given H (A/m)
$M_i(H)$	Magnetisation on the initial magnetisation curve for a given H (A/m)
$M_{irr}$	Irreversible component of magnetisation (A/m)
$M_{rev}$	Reversible component of magnetisation (A/m)
$M_s$	Saturation magnetisation (A/m)
$M_u(H)$	Magnetisation on the upward trajectory of the limiting hysteresis loop for a given H (A/m)
$m$	Magnetic moment (J/T)
$N$	Number of turns of a coil
$P_a$	Alternating anomalous loss (W)
$P_c$	Alternating core loss (W)
$P_e$	Alternating eddy current loss (W)
$P_{ea}$	Sum of alternating eddy current and anomalous losses (W)
$P_h$	Alternating hysteresis loss (W)
$p_a$	Average specific alternating anomalous loss over a cycle (W/kg)
$p_e$	Average specific alternating eddy current loss over a cycle (W/kg)
$p_h$	Average specific alternating hysteresis loss over a cycle (W/kg)
$P_r$	Specific core loss due to rotational component of an elliptically rotating magnetic field (W/kg)
$P_{ar}$	Specific rotational anomalous loss (W/kg)
$P_{er}$	Specific rotational eddy current loss (W/kg)
$P_{hr}$	Specific rotational hysteresis loss (W/kg)
$P_t$	Total specific core loss (W/kg)
$P_{ta}$	Total specific anomalous loss (W/kg)
$P_{te}$	Total specific eddy current loss (W/kg)
$P_{th}$	Total specific hysteresis loss (W/kg)
$R_a$	Equivalent resistance for alternating anomalous loss ( $\Omega$ )
$R_B$	Axis ratio of an elliptically rotating flux density
$R_{DC}$	DC resistance of an assumed eddy current path in a solid magnetic core ( $\Omega$ )
$R_e$	Equivalent resistance for alternating eddy current loss ( $\Omega$ )
$R_H$	Axis ratio of an elliptical rotating field strength

$R_w$	Winding resistance ( $\Omega$ )
$r_c$	Correction factor to $R_{DC}$ when anomalous loss is included
$r_J$	Correction factor to $R_{DC}$ when variation of eddy current density is considered
$T$	Time period, and $T=1/f$ (s)
$T_p$	Propagation time in the TLM method (s)
$T_r$	Retarding torque per unit volume due to $P_r$ (Nm/m <sup>3</sup> )
$t$	Time instant (s)
$V_L$	Voltage across an inductor or induced emf across the secondary coil of an annular ring sample (V)
$V_s$	Excitation voltage (V)
$W$	Energy (J)
$\delta$	Skin depth, and $\delta = \sqrt{\frac{2}{\sigma\omega\mu_r\mu_o}}$ (m)
$\Phi$	Magnetic flux (Wb)
$\gamma_{\alpha\beta}(H)$	Switching function of the elementary dipoles in the normal Preisach model of magnetic hysteresis
$\lambda$	Magnetic flux linkage (Wb)
$\mu(\alpha,\beta)$	Distribution function of the elementary magnetic dipoles in the normal Preisach model of magnetic hysteresis
$\mu_o$	Permeability of a vacuum (Tm/A)
$\mu_r$	Relative permeability
$\mu_\Delta$	Incremental permeability, and $\mu_\Delta = \frac{B_\Delta}{\mu_o H_\Delta}$
$\rho_m$	Mass density of material (kg/m <sup>3</sup> )
$\sigma$	Conductivity ( $\Omega^{-1}m^{-1}$ )
$\omega$	Angular frequency, and $\omega=2\pi f$ (rad/s)
$\chi_i$	Susceptibility of the initial magnetisation curve, and $\chi_i = \frac{dM_i}{dH}$
$\chi_{io}$	Initial susceptibility, and $\chi_{io} = \left. \frac{dM_i}{dH} \right _{H=0}$
$\chi_{ano}$	Gradient or susceptibility of anhysteretic magnetisation curve at the origin, and $\chi_{ano} = \left. \frac{dM_{anh}}{dH} \right _{H=0}$